ENGINEERING GRAPHICS SEMINAR

FOUR-DIMENSIONAL DESCRIPTIVE GEOMETRY

METRIC PROBLEMS: DISTANCES

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ABSTRACT

The paper presents the determination, in orthographic projections, of the true length of the distance between two given geometric elements in a four-dimensional space.
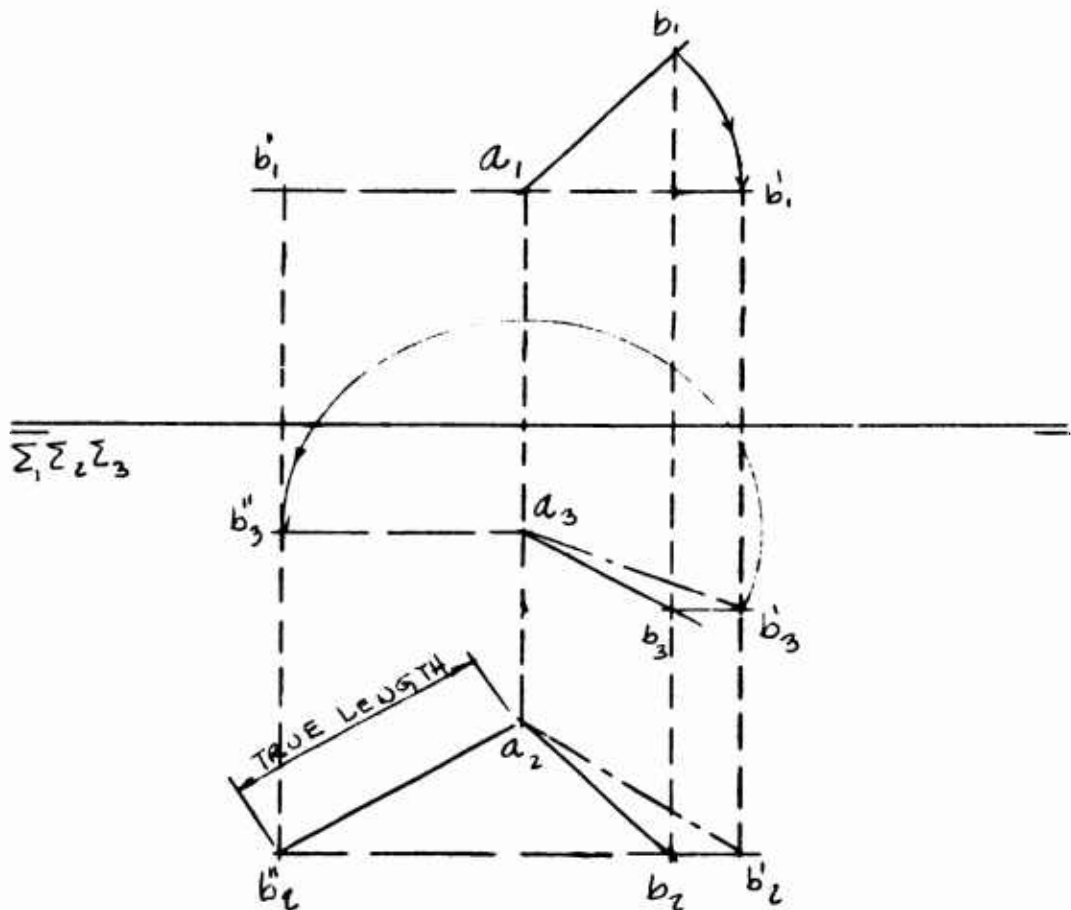
Problem No. 1.

"Determine the distance between two given points".

Solution

Figure 1 shows the determination of the true length of the line segment (ab). The descriptive method of rotation is applied.

Figure 1

Problem No. 2.

"Determine the distance from a point to a line".

Solution

Let (a) be the given point and (R) the given line, where we take two points, (b) and (c).

The simplest solution is to determine the true length of the sides of triangle (abc) and the height of it in respect to the vertice (a).

Problem No. 3.

"Determine the distance from a point to a plane".

Solution

Let (a) be the given point and $\propto$ the given plane, where we take three points, (b), (c), and (d).

The simplest solution is to determine the true length of the sides of the triangle (bcd) and of the line segments (ab, (ac), (ad). The distance sought is the height of pyramid of base (abc) and vertex (a).

Problem No. 4.

"Determine the distance between two parallel planes".

Solution

The two planes determine a 3-D space.

Let T be this space and α and β the two parallel planes, also parallel to Σ_1 - (if they are not parallel to Σ_1 apply a change of 3-D space of projection). Through a point (a) of α draw perpendicular to β and determine the foot of the perpendicular, point (b). The distance sought is the true length of (ab). Figure 2.

Problem No. 5.

"Determine the distance between two semi-parallel planes".

Solution

The planes belong to distinct 3-D spaces.

Let α be a plane of the 3-D space T and β a plane of the 3-D space Λ. Both planes are parallel to the 3-D space Σ_1.

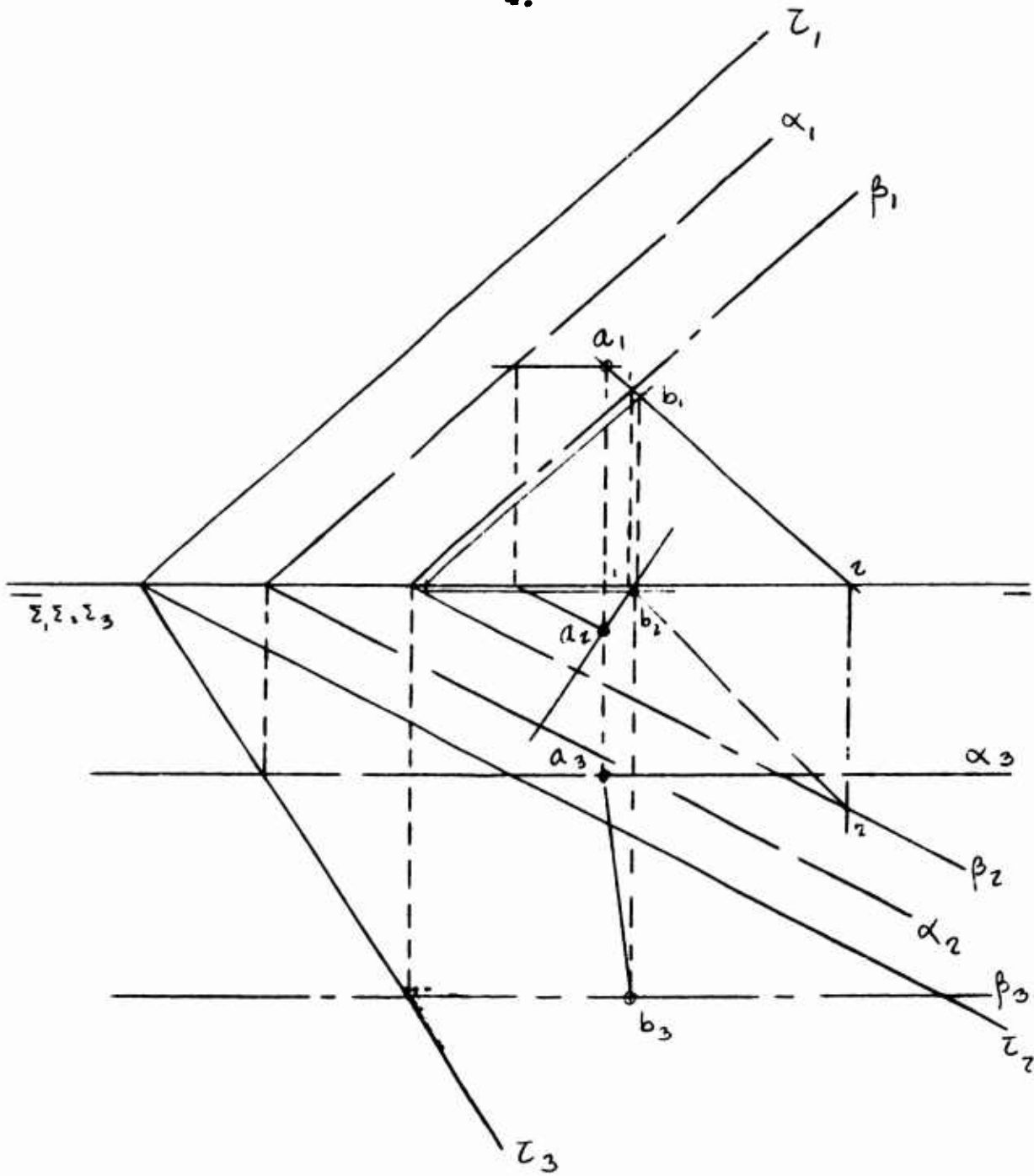**Figure 2**

Determine: (Figure 3)

a)   intersection of $\wedge$ and T , plane (abc);

b)   intersection of (abc) and α , line (kl);

c)   intersection of (abc) and β , line (mn).

The intersection of (mn) and (kl) is a point (o) common to α and β . But since these planes are parallel to Σ_1, the lines (mn) and (kl) are parallel. Therefore, point (o) is a point of the infinity and planes α and β are semi-parallel.

To determine the distance between the two planes, take a point (S) of (mn) and two points (v) and (L) of (kl) . The distance sought is the height of the triangle (svh) in respect to the vertice (S).
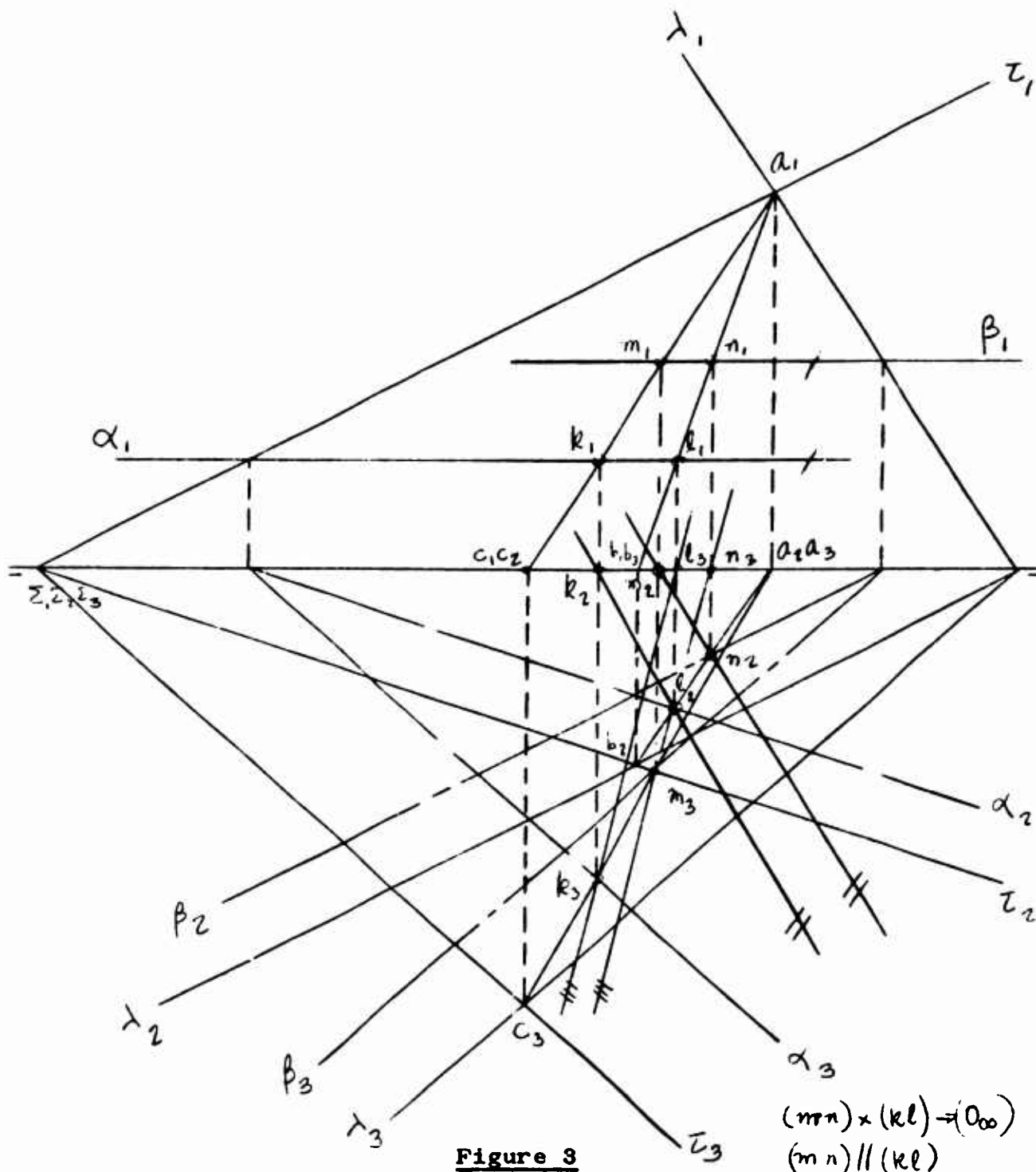
Problem No. 6.

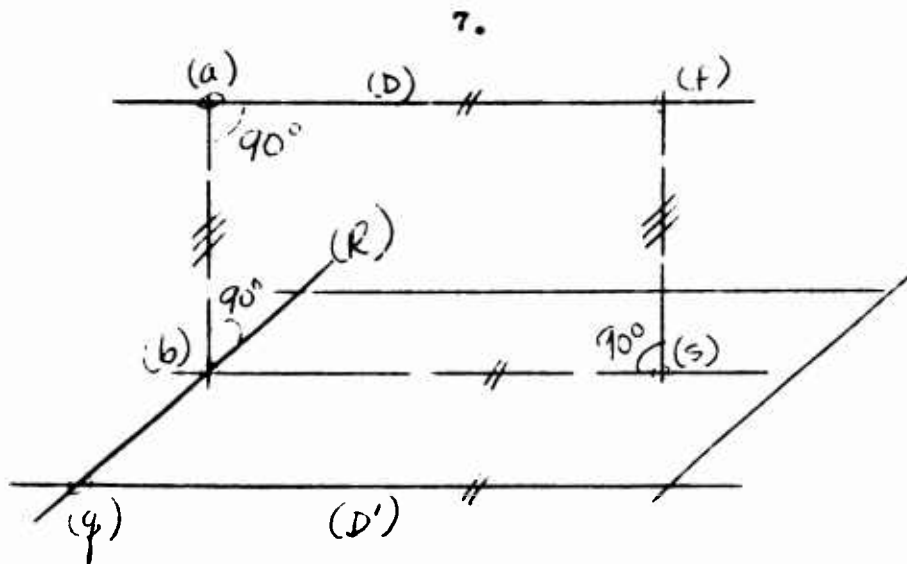"Determine the distance between two skew lines".

Solution

Let (R) and (D) be the two skew lines.   Through a point (q) of (R) draw (D') parallel to (D).   (Figure 4).

Figure 3

$$(mn) \times (kl) \rightarrow (0_\infty)$$
$$(mn) \parallel (kl)$$

GIVEN: (D) AND (R)

Figure 4

Since (R) and (D) determine a 3-D space T , in this 3-D space, through (p) draw (ps) perpendicular to the plane (R - D'), where (s) is the foot of the perpendicular.

Then, draw (sb) parallel to (D') - (b) belonging to (R) - and through (b) parallel to (ps). This parallel will be concurrent with (D) at a point (a).

The line segment (ab) is the distance between the two skew lines. It is their common perpendicular.

In Figure 5, we show the solution, in orthographic projections.
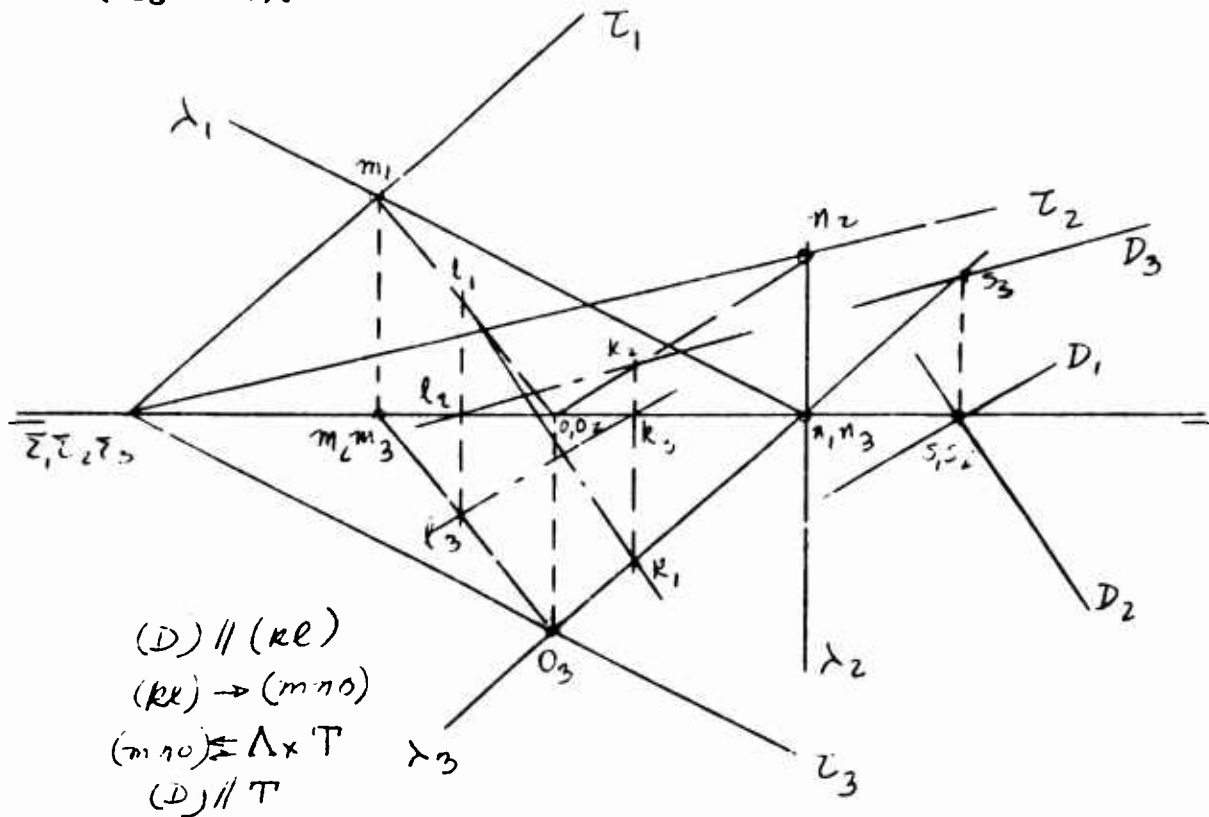
Figure 5

Problem No. 7.

"Determine the distance between a line and a 3-D space".

Solution

Naturally, the line and the 3-D space are assumed to be parallel.

Through the given line (D) consider a 3-D space $\bigwedge$ which intersects the given 3-D space T along a plane (mno). (Figure 6).

$(D) // (k\ell)$

$(k\ell) \rightarrow (mno)$

$(mno) \doteq \bigwedge \times T$

$(D) // T$

Figure 6

The distance from a point (a) of (D) to the plane (mno) is the distance from (D) to the 3-D space T .

Problem No. 8

"Determine the distance between a plane and a 3-D space".

Solution

The plane and the 3-D space are parallel.

Through a point (a) of the given plane α draw perpendicular to the given 3-D space T . The perpendicular and the plane α determine a 3-D space Λ . Find the intersection of T and Λ , plane β . The planes α and β are parallel and the distance between α and β is the distance between α and T . (Figure 7).

Problem No. 9.

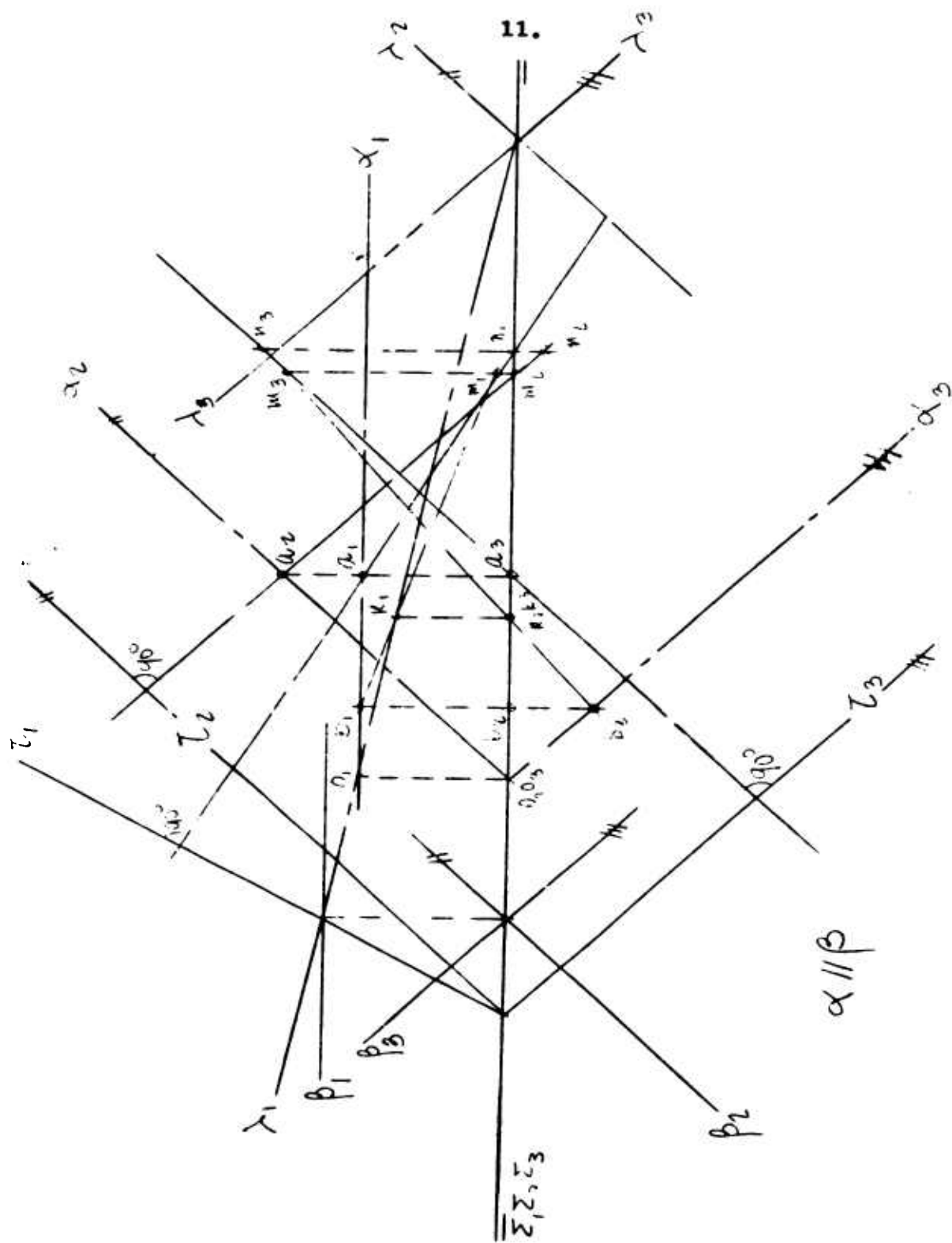"Determine the distance between two parallel 3-D spaces".

11.

$\alpha \parallel \beta$

Figure 7